

Mon. Not. R. Astron. Soc. **000**, 000–000 (2000)

Evolution of the cosmological density distribution function

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18 January 2000

ABSTRACT

We present a new calculation for the evolution of the 1-point Probability Distribution Function (PDF) of the cosmological density field based on an exact statistical treatment. Using the Chapman–Kolmogorov equation and second-order Eulerian perturbation theory we propagate the initial density distribution into the nonlinear regime. Our calculations yield the moment generating function, allowing a straightforward derivation of the skewness of the PDF to second order. We find a new dependency on the initial perturbation spectrum. We compare our results with other approximations to the 1-pt PDF, and with N-body simulations. We find that our distribution accurately models the evolution of the 1-pt PDF of dark matter.

Key words: Cosmology: theory – large-scale structure of Universe

1 INTRODUCTION

The standard scenario for the formation of structure in the Universe is via the gravitational amplification of primordial random Gaussian fluctuations generated in the Early Universe during an Inflationary phase. An attractive feature of this scenario is its predictive power in determining the history of mass perturbations from initial times up until the present day. In principle this should allow one to compare observation with theory. But while detailed predictions at high redshift for the Cosmic Microwave Background (CMB) are possible due to their linearity and dependence on well tested laboratory physics, predictions at lower redshift are complicated by nonlinear gravitational evolution and the physics of galaxy formation. One of the goals of cosmology is accounting for the statistical evolution of mass and galaxies up to the present day.

In this paper we focus on the properties of the one-point density distribution function of matter. Interest in the distribution functions of cosmological fields has grown as they encode a great deal of information on initial conditions, gravitational evolution and galaxy bias. However the challenge is to separate out each of these effects.

The evolution of the one-point distribution function has been computed by a number of authors using a variety of techniques here we present a new approach, based on propagating the distribution function by the Chapman–Kolmogorov equation, incorporating nonlinear evolution to second order. In a further paper we shall develop the formalism to include the effects of galaxy biasing and redshift-space distortions (Watts & Taylor 2000).

The 1-pt PDF is a useful quantity in cosmology. While reasonably straightforward to measure (see Hamil-

ton 1985, Alimi et al 1990, Szapudi, Szalay & Boschán 1992, Gaztañaga 1992, 1994, Bouchet et al 1993, Szapudi, Meiksin & Nichol 1996, Kim & Strauss 1998) the 1-pt PDF embodies the full hierarchy of correlation functions and its measurement ensures that physical constraints such as the Lyapunov inequalities for the moments ($\langle x^{a+b} \rangle \geq \langle x^a \rangle \langle x^b \rangle$, for any random variable x) are automatically satisfied. This is not necessarily the case for direct measurement of the moments. In addition the PDF offers a convenient way of dealing with Poisson sampling in the galaxy distribution.

From a theoretical perspective, the 1-pt PDF is a convenient quantity to calculate if it is initially Gaussian. A number of different methods have been developed to calculate its evolution. Fry (1985) first suggested calculating the probability function by applying a hierarchical series suggested by the BBGKY equations to solve for the moment generating function. Bernardeau (1992) derived an exact expression for the evolution of the moment generating function. The generalisation to top-hat filtered density and velocity fields is found in Bernardeau (1994), while Bernardeau (1996) studied the 2-point cumulants and the PDF. However so far a generalisation of these results to include bias and redshift-space distortions in the PDF has not been presented.

An approximation for the PDF can also be constructed from the first few cumulants, in the limit of small variance, by the Edgeworth expansion (Juszkiewicz et al., 1995 and Bernardeau & Kofman, 1995), where the cumulants have been derived directly from Eulerian perturbation theory (Bouchet et al., 1992) or Lagrangian perturbation theory (Bouchet et al. 1995). This can be extended to redshift-space using the Lagrangian perturbation calculations of Hivon et al. (1995) for the skewness. Colombi et al. (1997)

introduced an extended perturbation theory, where the results of perturbation theory are allowed extra freedom and extrapolated to the nonlinear regime.

As well as Eulerian perturbation theory other approximations have also been applied. Kofman et al (1994) and Bernardeau & Kofman (1995) derived the PDF in the Zel'dovich approximation, while Hui, Kofman & Shandarin (1999) extended this to include the effects of redshift-space distortions.

A more phenomenological approach was taken by Coles & Jones (1991) who approximated the properties of the PDF by a lognormal distribution while Colombi (1994) suggested an Edgeworth expansion about the lognormal distribution.

In this letter we apply second-order perturbation theory to an initially Gaussian distributed density field to calculate the exact second-order characteristic function. We then numerically inverse Fourier transform this to yield the 1-pt pdf. Our method therefore treats the propagation of probabilities exactly.

The letter is organised as follows. In Section 2 we calculate the evolution of the cosmological probability distribution function and discuss the transformation to a discrete count distribution. In Section 3 we compare our results with the results of N-body simulations and with other distributions in Section 4. Conclusions are presented in Section 5. We begin by describing second-order perturbation theory and calculating the probability distribution function.

2 THE COSMOLOGICAL PROBABILITY DISTRIBUTION FUNCTION

2.1 Second-order perturbation theory

Before shell-crossing, and in a spatially flat universe, the Eulerian density field, $\delta(\mathbf{x}, t)$, can be expanded in a series of separable functions;

$$\delta(\mathbf{x}, t) = \sum_{n=1}^{\infty} \delta_n(\mathbf{x}, t) = \sum_{n=1}^{\infty} D_n(t) \varepsilon_n(\mathbf{x}), \quad (1)$$

where ε_n is an n^{th} -order time-independent density field and D_n is an n^{th} -order universal growth function. It is a special property of this expansion that the spatial and time components are separable. In a universe with spatial curvature this expansion is only possible to second-order.

The growth of perturbations in the nonlinear regime can be calculated by solving the continuity, Euler and Poisson equations sequentially for each order in the perturbation expansion. To second-order the density field can be derived from linear quantities by the relation (Peebles 1980, Bouchet et al 1992)

$$\delta = \delta_1 + \frac{1}{3}(2 - \kappa)\delta_1^2 - \boldsymbol{\eta} \cdot \mathbf{g} + \frac{1}{2}(1 + \kappa)E^2, \quad (2)$$

where

$$\boldsymbol{\eta}(\mathbf{x}, t) = \nabla \delta_1(\mathbf{x}, t) \quad (3)$$

is the gradient of the linear density field and

$$\mathbf{g}(\mathbf{x}, t) = -\nabla \nabla^{-2} \delta_1(\mathbf{x}, t) \quad (4)$$

is the linear peculiar gravity field^{*} where ∇^{-2} is the inverse Laplacian. The trace-free tidal tensor is given by

$$E_{ij}(\mathbf{x}, t) = \nabla_i \nabla_j \nabla^{-2} \delta_1(\mathbf{x}, t) - \frac{1}{3} \delta_{ij}(\mathbf{x}, t) \delta_{ij}. \quad (5)$$

Equation (2) is the general result for an arbitrary cosmology where[†] $\kappa = D_2/D_1^2 \approx -3/7\Omega^{-2/63}$ (Bouchet et al 1992, see also Catelan et al. 1995), and $\delta_1(\mathbf{x}, t) = D_1(t)\varepsilon_1(\mathbf{x})$, where $D_1(t) \approx (1+z)^{-\Omega^{0.6}}$ is the linear growth function (Peebles 1980).

2.2 The distribution function of initial fields

In order to calculate the second-order density distribution function, $P(\delta)$, we must first calculate the joint probability of each of the fields in equation (2), $P(\delta_1, \boldsymbol{\eta}, \mathbf{g}, \mathbf{E})$. Defining the parameter vector $\mathbf{y} = (\delta_1, \boldsymbol{\eta}, \mathbf{g}, \mathbf{E})$ the joint distribution function is given by the multivariate Gaussian

$$P(\mathbf{y}) = \frac{1}{((2\pi)^n |\det \mathbf{C}|)^{1/2}} \exp\left(-\frac{1}{2} \mathbf{y}^t \mathbf{C}^{-1} \mathbf{y}\right), \quad (6)$$

where

$$\mathbf{C} = \langle \mathbf{y} \mathbf{y}^t \rangle \quad (7)$$

is the 12×12 covariance matrix that gives the degree of correlation between each of the components of \mathbf{y} . The elements of the covariance matrix are

$$\begin{aligned} \langle \delta^2 \rangle &= \sigma_0^2, & \langle \eta_i \eta_j \rangle &= \frac{1}{3} \sigma_1^2 \delta_{ij}, \\ \langle g_i g_j \rangle &= \frac{1}{3} \sigma_{-1}^2 \delta_{ij}, & \langle \eta_i g_j \rangle &= \frac{1}{3} \sigma_0^2 \delta_{ij}, \\ \langle E_{ij} E_{kl} \rangle &= \frac{1}{15} \sigma_0^2 \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{kl} \right), \end{aligned} \quad (8)$$

with the remaining entries in the covariance matrix zero. These variances are defined as

$$\sigma_n^2 = D^2(t) \int_0^\infty \frac{dk}{2\pi^2} k^{2+2n} P(k), \quad (9)$$

where $P(k)$ is the initial power spectrum of density perturbations. The components of this distribution are

$$\mathbf{y}^t \mathbf{C}^{-1} \mathbf{y} = \frac{\delta^2}{\sigma_0^2} + \frac{3}{(1 - \gamma_\nu^2)} \left(\frac{\eta^2}{\sigma_1^2} + \frac{g^2}{\sigma_{-1}^2} - 2\gamma_\nu \frac{\boldsymbol{\eta} \cdot \mathbf{g}}{\sigma_1 \sigma_{-1}} \right) + 15 \frac{E^2}{\sigma_0^2} \quad (10)$$

and

$$\det \mathbf{C} = \frac{20}{3105^3 \pi^4} \sigma_0^{12} \sigma_1^6 \sigma_{-1}^6 (1 - \gamma_\nu^2)^3. \quad (11)$$

The correlation parameter, γ_ν , is defined as

$$\gamma_\nu = \frac{\sigma_0^2}{\sigma_1 \sigma_{-1}}, \quad (12)$$

providing a measure of the correlation between the linear velocity and density gradient fields. If we assume a power-law power spectra with a Gaussian cut-off, $P(k) \propto k^n e^{-k^2 R^2}$,

^{*} In this paper we define $4\pi G \rho_0 = 3/2\Omega H^2 = 1$ and the expansion parameter $a(t) = 1$, since our final distribution function will be dimensionless.

[†] We define κ following Bouchet et al. (1995) and Hivon et al. (1995), but note that this differs from the definition of Bouchet et al. (1992) who define $\kappa \approx 2/14\Omega^{-2/63}$

where n is the spectral index and R some arbitrary length scale, then

$$\gamma_\nu = \frac{n+1}{n+3}, \quad (13)$$

with the constraint $n > -1$ to insure convergence of the velocity field. The correlation parameter must be positive definite since under gravity matter will be displaced from low- to high-density regions. The special case of $n \rightarrow -1$ for a power-law initial spectra results in $\gamma_\nu = 0$ because the velocity field diverges on small scales. In fact the diverging velocities form an incoherent Gaussian random field which is uncorrelated with the density field (Taylor & Hamilton 1996).

2.3 Propagation of the density distribution function

The distribution function can be propagated to later times by the Chapman–Kolmogorov equation,

$$P(\mathbf{x}) = \int d\mathbf{y} W(\mathbf{x}|\mathbf{y})P(\mathbf{y}) \quad (14)$$

where $W(\mathbf{x}|\mathbf{y})$ is the transition probability from \mathbf{x} to \mathbf{y} . In the case of a deterministic process, such as the one considered here, the transition probability reduces to a delta function restricting the number of possible paths of evolution to one. This transition probability is given by

$$W(\delta|\mathbf{y}) = \delta_D\left[\delta - \delta_1 - \frac{1}{3}(2 - \kappa)\delta_1^2 + \boldsymbol{\eta} \cdot \mathbf{g} - \frac{1}{2}(1 + \kappa)E^2\right] \quad (15)$$

Inserting this into equation (14) we find

$$P(\delta) = \langle \delta_D(\delta - \delta(\mathbf{y})) \rangle_{\mathbf{y}}. \quad (16)$$

where $\delta(\mathbf{y})$ is the right hand side of equation (2). Hence for deterministic transitions the Chapman–Kolmogorov equation becomes the expectation value of the delta function. This is a well-known result from probability theory (eg van Kampen 1992).

Fourier transforming the delta function we find

$$P(\delta) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} dJ \mathcal{G}(J) \exp(iJ\delta) \quad (17)$$

where

$$\begin{aligned} \mathcal{G}(J) &\equiv \int_{-\infty}^{\infty} d\delta P(\delta) \exp(-iJ\delta) \\ &= \langle \exp(-iJ\delta(\mathbf{y})) \rangle_{\mathbf{y}} \end{aligned} \quad (18)$$

is the characteristic function. In the second line we have used equation (16) to write the characteristic function as an expectation over all the linear fields, \mathbf{y} . This expression reduces to a set of multivariate Gaussian-type integrals that we can easily evaluate yielding

$$\mathcal{G}(J) = \Theta(J) \exp\left[-\frac{J^2 \sigma_0^2}{2(1 + i\alpha_1 J)}\right] \quad (19)$$

where

$$\Theta(J) = (1 + i\alpha_1 J)^{-1/2} (1 + i\alpha_2 J)^{-5/2} (1 - i\alpha_3 J + \alpha_4 J^2)^{-3/2} \quad (20)$$

and where the α coefficients are given by

$$\alpha_1 = \frac{2}{3}(2 - \kappa)\sigma_0^2, \quad \alpha_2 = \frac{2}{15}(1 + \kappa)\sigma_0^2,$$

Figure 1. The evolution of the 1-pt distribution function, $P(\delta)$ calculated from second-order perturbation theory. The variances are $\sigma_0 = 0.15, 0.20, 0.25, 0.3$. In the lower plot we use logarithmic axis to emphasize the tails of the distribution.

$$\alpha_3 = \frac{2}{3}\sigma_0^2, \quad \alpha_4 = \frac{1}{9} \frac{(1 - \gamma_\nu^2)}{\gamma_\nu^2} \sigma_0^4 \quad (21)$$

The probability distribution can then be found by numerically integrating equation (17). Equations (19), (20) and (21) are the central results of this letter.

The characteristic function for δ can be separated into characteristic functions for each term in the summation in equation (2). This is to be expected, since the distribution of a summation of random variables is equal to the convolution of their individual distributions, and so the characteristic functions just multiply. We expect this to be true for all orders of the perturbation series. But rather than converging to a Gaussian distribution, as suggested by a naive application of the Central Limit Theorem, the final distribution is driven away from it by the correlations induced by gravity.

Figure 1 shows the evolved density distribution function for a range of σ_0 . We use linear axis for the top plot emphasising the peak and logarithmic axis in the lower plot emphasising the tail of the distribution. As expected, the shape of the distribution is very nearly Gaussian when the variance is small, becoming very rapidly non-Gaussian for higher values. For high variances the probability density does not drop to zero at $\delta = -1$, since in second-order perturbation theory the density field can be negative, generating non-vanishing regions with $\delta \leq -1$. This is also true of linear theory where there are always negative density regions for Gaussian initial conditions. This is a feature of Eulerian distribution functions. Those calculated in Lagrangian perturbation theory, or the lognormal, have positive definite densities at all times.

Figure 2. The variation in the peak (upper) and tail (lower) of the 1-point density distribution function as a function of the correlation parameter, γ_ν . The values used were $\gamma_\nu = 0.65$ (solid line), 0.55 (dashed), 0.45 (dot-dashed) and 0.35 (dotted). The effect is small for CDM-type initial power spectra where the correlation parameter is in the range $0.55 < \gamma_\nu < 0.65$ over a wide range of scales.

In Figure 2 we demonstrate the effects of the correlation parameter, γ_ν , on the PDF. The main effects are an increase in volume of underdense regions with a corresponding decrease in extremely underdense regions when γ_ν is high. For low γ_ν , the underdensities are smaller and deeper. The physical reason for this is that the $\eta.g$ term in second-order perturbation theory deals with the evacuation of the voids, rather than the amplification of peaks. When γ_ν is low this term is weakened and voids tend to be smaller and deeper, as they would be if the linear field were extrapolated. Increasing this correlation widens the voids, but makes them shallower to help satisfy the $\delta \geq -1$ constraint. This effect is small for CDM-type initial power spectra where the correlation parameter is in the range $0.55 < \gamma_\nu < 0.65$ over a wide range of scales.

2.4 Skewness from the characteristic function

Since our characteristic function is correct to second-order a useful check is to calculate the variance and skewness to compare with previous estimates. Taking the derivatives of the characteristic function with respect to J and setting J equal to zero yields the moments of the evolved density distribution function,

$$\frac{\partial^n}{\partial [iJ]^n} \mathcal{G}(J=0) = \langle \delta^n \rangle. \quad (22)$$

In the literature it is common to express the moments in the form of the moment parameters

$$S_n = \langle \delta^n \rangle_c / \langle \delta^2 \rangle^{n-2} \quad (23)$$

where $\langle \delta^n \rangle_c$ is the connected or irreducible part of $\langle \delta^n \rangle$. The irreducible moments, or cumulants, can be generated by

$$\frac{\partial^n}{\partial [iJ]^n} \ln \mathcal{G}(J=0) = \langle \delta^n \rangle_c \quad (24)$$

From this we can calculate the second and third order connected moments of our distribution function;

$$S_2 = 1 \quad (25)$$

and

$$S_3 = 2(2 - \kappa) \xrightarrow{\Omega \rightarrow 1} \frac{34}{7} \quad (26)$$

to lowest order, reproducing the results of Peebles (1980). Hence our PDF leads to the correct second-order skewness. The intrinsic effects of the shape of the power spectrum via the correlation coefficient γ_ν are to higher order.

2.5 The discrete distribution function

In reality the density field of galaxies is not a continuous function, but a discrete distribution. To account for this it is usual to assume Poisson sampling of the continuous density field as a crude approximation to galaxy formation processes. Any variation is attributed to biased galaxy formation, which modulates the underlying function. We shall consider this elsewhere (Watts & Taylor, 2000).

The continuous density distribution function can be transformed to a discrete form by the expectation

$$P(n) = \frac{\bar{n}^n}{n!} \langle (1 + \delta)^n e^{\bar{n}(1+\delta)} \rangle_\delta, \quad (27)$$

where \bar{n} is the mean galaxy count and the expectation is taken with respect to the nonlinear density distribution. Expanding this in terms of the characteristic function and taking the expectation we find

$$P(n) = \int_{-\infty}^{\infty} dJ G(J) \left(1 - \frac{iJ}{\bar{n}}\right)^{-n-1} \exp(-iJ) \quad (28)$$

In the limit $\bar{n} \rightarrow \infty$ this returns the continuum distribution where $\delta = n/\bar{n} - 1$. We note that again that the discrete distribution has a non-vanishing value at $n = 0$, $P(n=0)$, since the probability of finding a no galaxies within a cell is finite. This is the Void Probability Function (White 1979).

It is also useful to have the generating function for the discrete moments of this distribution, $G_n(k)$. Following some straightforward calculation we find that

$$G_n(k) = G[i\bar{n}(e^{-ik} - 1)] \exp \bar{n}(e^{-ik} - 1). \quad (29)$$

The discrete moments can then be found by differentiating $G_n(k)$;

$$\langle \delta^m \rangle = \frac{\partial^m G_n(k=0)}{\partial (ik)^m} \quad (30)$$

Again the connected moments can be found by differentiating $\ln G(k)$ with respect to ik .

Figure 3. Comparison of the second-order PDF (solid line) with the results of N-body simulations (points). The variances are $\sigma_0 = 0.2$ (open triangles) and 0.3 (filled squares). Also plotted are the corresponding distribution functions for the lognormal model (dotted) and Edgeworth expansion (dashed).

3 COMPARISON OF RESULTS WITH N-BODY SIMULATIONS

In this section we show a comparison between our theoretical PDF and the counts in cells PDF found from cosmological n-body simulations. We used a version of Hugh Couchman's (1991) Adaptive P³M code altered by Peacock & Dodds (1994) to allow simulations of low density open and flat universes.

The simulation volume was a periodic cube of comoving side $200 h^{-1} \text{Mpc}$ containing 100^3 particles. We chose a CDM-type linear power spectrum of Gaussian initial perturbations (Bardeen et al. 1986), normalised to match the observed abundance of clusters with linear variance given by $\sigma_8 = 0.6\Omega^{-0.53}$ (Viana & Liddle 1996). The simulation was carried out on a 128^3 Fourier mesh.

To measure the PDF from the numerical simulations we smoothed the discrete particle distribution with a Gaussian filter of radius R . The PDF was then found from the smoothed density field evaluated on a 128^3 grid. Since the effective radius of the binning grid was much smaller than that of the filter radius we expected negligible contribution to the smoothing from it.

The choice of variance to use in our model is slightly ambiguous, given that we do not treat smoothing exactly. Hence it is better to match variances rather than smoothing scales. However the variance in second-order perturbation theory is the same as in linear theory so we could use either the linear input variance from the simulations, or take the measured nonlinear variance when making a comparison. In

practice we found that all of the models provided a better fit if the nonlinear variances were used. This is a well known effect and is the basis of the extended perturbation theory approach of Colombi et al (1997).

Figures (3, top) and (3, bottom) show the results of numerical simulations (points) alongside our second-order PDF (solid line), the lognormal model (Coles & Jones 1991, dotted) and the Edgeworth expansion (Juszkiewicz et al. 1995, Bernardeau & Kofman 1995; dashed). Each plot shows the PDF for two different variances, $\sigma_0 = 0.2$, smoothed on a scale of $R = 20 h^{-1} \text{Mpc}$ (open circles) and 0.3 (filled squares) smoothed on $13.5 h^{-1} \text{Mpc}$.

The effects of shot noise were taken into account as shown in Section 2.3 with the mean particle density \bar{n} chosen to coincide with the mean particle count in a boxes with side $l = \sqrt{5}R$. Since \bar{n} was a very large number in all cases, shot noise effects were extremely small. This would not be the case in a real galaxy survey with lower number counts. The remaining factor controlling the shape of the theoretical PDF was γ_ν . For linear CDM power spectra the calculated γ_ν was approximately 0.55 for a wide range of smoothing radii and it was this value that was used to prepare all the plots in this section.

Error bars on the N-body data points are the standard deviation over 5 independent simulations with identical cosmological parameters.

The linear axes of figures (3, top) shows the accuracy around the peak of the distributions while the logarithmic scale of figure (3, bottom) shows the tails of the distributions. At low variance ($\sigma_0 = 0.2$) we find very good agreement between all of the models and the simulations. Overall we found that the lognormal model fitted the simulations extremely well for all values of the variance and around both the peak and the tails. As has been remarked elsewhere we regard this as something of a fluke (Bernardeau & Kofman 1995), and that the lognormal makes a very useful fitting function.

Our second-order PDF fits the peak fairly accurately, but tends to be slightly too skewed. This effect becomes greater at higher variance. The tails of the distribution match the N-body simulations rather well, although at high variances and low densities the distribution is too high. This is due to the real distribution being constrained to go to zero at zero density, whereas, as we have already remarked the second-order PDF is not. In comparison the Edgeworth expansion, taken to second order (Juszkiewicz et al. 1995, Bernardeau & Kofman 1995)

$$P_E(\delta) = \left[1 + \frac{1}{6} S_3 \sigma_0 H_3(\delta/\sigma_0) \right] \frac{e^{-\delta^2/2\sigma_0^2}}{\sqrt{2\pi}}, \quad (31)$$

where $H_n(x)$ is a Hermite polynomial, is also slightly too skewed and tends to undershoot the high density tail compared to the simulations. At large variance the expansion breaks down and the Edgeworth distribution develops a wiggle on the high density tail. In fact the Edgeworth expansion fares badly even when taken to third order because the unphysical wiggles are exaggerated and the low density tail becomes more rapidly negative.

Figure 4. Comparison with other distributions. We normalise all the models to the lognormal distribution ($P_{\text{lnorm}}(\delta)$; dotted line), since we find that this best fits our N-body simulation results over a wide range of parameter values. The plot shows the ratio $(P(\delta) - P_{\text{lnorm}}(\delta))/P_{\text{lnorm}}(\delta)$ for our second-order calculation (solid line), the Zel'dovich approximation (Kofman et al. 1994; dot-dashed line) and the Edgeworth expansion (dotted line). The variance used for the comparison was $\sigma_0 = 0.25$.

4 COMPARISON WITH OTHER APPROXIMATIONS

Finally we compare our PDF with other approximations for the 1-point density distribution. In figure 4 we show three distributions, our second-order PDF calculation (solid line), the Zel'dovich approximation (Kofman et al. 1994; dot-dashed line) and the Edgeworth expansion (dotted line) plotted relative to the lognormal distribution. Our choice of the lognormal distribution as the point of normalisation is based on the agreement we find between it and our N-body simulations.

Over the range $0.4 < \delta < 1.5$ all of the models agree to within a few percent, with the exception of the second-order Edgeworth expansion. But for densities below this regime, our second-order PDF overpredicts because the positive density condition is only weakly met. Both the Zel'dovich and Edgeworth models underpredict the probability of low density regions. Above the regime of good agreement our model again overpredicts, again because we over-extrapolate the high density evolution, but not as much as the Zel'dovich approximation, where caustic formation tends towards a too high distribution. In contrast to both models the Edgeworth approximation underpredicts the high density regions and develops a large wiggle in the positive density regime.

5 SUMMARY

We have presented a new method for calculating the non-linear evolution of the 1-point density distribution function using the Chapman–Kolmogorov equation to propagate the probability distribution, and second-order perturbation theory to evolve the density field. This has the advantage over

other methods that the resultant probability distribution must be positive definite, and can be readily extended to include the effects of Eulerian deterministic or stochastic bias and redshift space distortions. The main disadvantages of our method are that it is not obvious how to include the effects of smoothing in the final distribution and it is difficult to see how one could extend the evolution of the density field to higher order. However, despite these problems we find extremely good agreement with numerical simulations and that our distribution is an improvement on other approximations. Since we derive the characteristic function the moments and cumulants (connected moments) of the field are easily derived, as is the distribution of any local transformations of the density field. The formalism we have introduced can also be used to calculate more complicated 1- and 2-point distributions of cosmologically interesting fields.

Acknowledgements

PIRW thanks the PPARC for a postgraduate grant. ANT thanks the PPARC for a postdoctoral fellowship.

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